Pressure-Geometry Scaling for Chemical Laser Performance

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Introduction

In high energy chemical lasers, among the most important performance parameters are power flux, lasing mode length, and specific power. It is shown in the present analysis that, by utilizing pressure-geometry scaling, a direct tradeoff between power flux and lasing mode length can be made while maintaining a desired specific power. This result greatly simplifies experimental and theoretical performance evaluation tasks by reducing a three-parameter family of variables to a two-parameter one. Scaling of other properties such as gain and intensity is also indicated.

The scaling analysis is carried out by application of the flowfield aerokinetics-radiation conservation equations and constitutive relations. The scaling results are verified by numerical solution of the governing equations.

Transport Equations

The conservation equations for compressible reacting flow are well known (e.g., Refs. 1 and 2). For scaling purposes, the equations for steady two-dimensional flow (transient and three-dimensional effects are indicated later) are written in terms of the following nondimensional variables

$$U=u/u_0, \quad V=v/u_0 \tag{1}$$

$$H=h/h_0, \quad E=(u^2+v^2)/2h_0 \tag{2}$$

$$\tau^*=\tau/\rho_0 u_0^2, \quad q^*=q/\rho_0 u_0 h_0, \quad j^*=j/\rho_0 u_0 \tag{3}$$

$$p^*=p/p_0, \quad \rho^*=\rho/\rho_0 \tag{4}$$

$$\xi=x/L, \quad \eta=y/L \tag{5}$$

where the subscript 0 denotes a reference value, and L is a characteristic length. The velocity components u and v correspond to the axial direction x and lateral direction y, respectively; h, ρ, and p denote specific enthalpy, density, and pressure; and τ, q, and j denote stress, heat flux, and mass diffusion flux, respectively.

The conservation equations may then be written as follows.

Mass:

$$\frac{\partial}{\partial\xi}(\rho^*U)+\frac{\partial}{\partial\eta}(\rho^*V)=0 \tag{6}$$

Axial momentum:

$$\frac{\partial}{\partial\xi}(\rho^*U^2-\tau_{xx}^*)+\frac{\partial}{\partial\eta}(\rho^*VU-\tau_{yx}^*)+\frac{p_0}{\rho_0 u_0^2}\frac{\partial p^*}{\partial\xi}=0 \tag{7}$$

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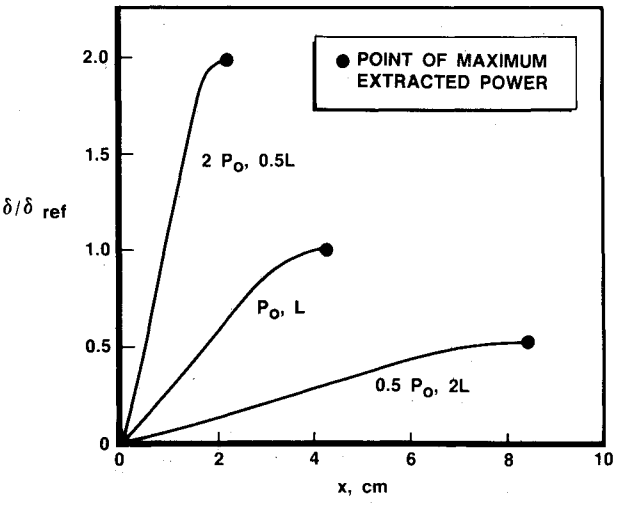

Fig. 1 Variation of normalized power flux with distance from cavity inlet.

Lateral momentum:

$$\frac{\partial}{\partial\xi}(\rho^*UV-\tau_{xy}^*)+\frac{\partial}{\partial\eta}(\rho^*V^2-\tau_{yy}^*)+\frac{p_0}{\rho_0 u_0^2}\frac{\partial p^*}{\partial\eta}=0 \tag{8}$$

Energy:

$$\frac{\partial}{\partial\xi}[\rho^*U(H+E)+q_x^*-\tau_{xx}^*U-\tau_{xy}^*V]$$

$$+\frac{\partial}{\partial\eta}[\rho^*V(H+E)+q_y^*-\tau_{yx}^*U-\tau_{yy}^*V]=\frac{L}{\rho_0 u_0 h_0}Q \tag{9}$$

Species:

$$\frac{\partial}{\partial\xi}(\rho^*Uc_i+j_{ix}^*)+\frac{\partial}{\partial\eta}(\rho^*Vc_i+j_{iy}^*)=\frac{L}{\rho_0 u_0}\Phi_i \tag{10}$$

where c denotes the mass fraction, and the subscript i refers to species i. The Q and Φ_i are energy and species source terms, respectively.

For simplicity, only the scaling between pressure level and geometric size is considered. That is, the laser nozzle/cavity hardware sets of different sizes are considered to have the same flow inlet temperature and composition. Also, the use of only one characteristic length implies that the hardware sets are geometrically similar.

The nondimensional stresses τ^* are given by the product of Re^{-1} and the nondimensional velocity gradients, where Re is the Reynolds number defined by

$$Re=\rho_0 u_0 L/\mu \tag{11}$$

and μ is the viscosity. With temperature, composition, and geometry being similar between two scales, the thermal properties, transport coefficients, and Mach number (hence u_0) will be similar as well. Hence, by imposing the scaling relation of

$$p_0 L=\text{const} \tag{12}$$

and noting that ρ_0/p_0 is a constant, one obtains similarity in Re.

The nondimensional heat fluxes q^* and mass diffusion fluxes j_i^* are functions of only Re, Pr, Sc_i, and the nondimensional enthalpy and concentration gradients, where Pr and Sc are the Prandtl and Schmidt numbers, respectively,

which are similar as a consequence of similarity in thermal properties.

Examination of Eqs. (6-10) then indicates that all nondimensionalized variables, except for the source terms Q and Φ_i yet to be discussed, become independent of scale. That is, the flowfields of two scaled configurations are similar if condition (12) is imposed. This is simply a consequence of the Reynolds principle of similarity. The more subtle and significant factors of concern to chemical laser performance scalability are the energy and species source terms Q and Φ_i, which are treated in the following section.

Chemical Kinetics and Radiation

The energy and species source terms are given by[3]

$$Q = -\sum_i \alpha_i I_i \tag{13}$$

$$\Phi_i = \dot{w}_i + \frac{\alpha_i I_i}{\epsilon_i} - \frac{\alpha_{i-1} I_{i-1}}{\epsilon_{i-1}} \tag{14}$$

where $\dot{w}_i$, α_i, I_i, and ϵ_i denote species chemical production rate, radiative gain, radiative intensity, and photon energy, respectively, of species i. For the nonlasing species, $\alpha_i = 0$. For the lasing species, the subscript i denotes any combination of vibrational level v and rotational state J, and the subscript $i+1$ denotes a vibrational level one higher (and rotational state one lower) than i.

The species production for a given chemical reaction is given by

$$\dot{w}_i = (\nu_i'' - \nu_i') \left[k_f \prod_i (\rho c_i / m_i)^{\nu_i'} - k_b \prod_i (\rho c_i / m_i)^{\nu_i''} \right] \tag{15}$$

where k_f and k_b are the forward and backward reaction rate constants, respectively, ν_i' and ν_i'' the stoichiometric coefficients, and m the molecular weight. For bimolecular reactions, $\dot{w}_i$ is proportional to ρ^2. Termolecular reactions are generally negligible for low pressure chemical lasers, and it is this fact that allows chemical reactions to be scalable by Eq. (12).

The gain is given by

$$\alpha_i = \epsilon_i \rho (S_{i+1} c_{i+1} - S_i c_i) G_i / m_i \tag{16}$$

where S and G are a line shape parameter and the Voigt function, respectively. The representations of S and G are quite lengthy.[3] For scaling purposes, it is sufficient to note that S is a function of temperature only, while G depends on temperature, pressure, and composition. Under low-pressure conditions, say <20 Torr, G is nearly independent of pressure, and α becomes a linear function of ρ.

The remaining variable of concern in the conservation equations is the radiation intensity, which is coupled to the optical field equations. For a Fabry-Perot cavity, the intensity may be given by a laser rate equation of the form[3]

$$\frac{dI_i}{d\xi} = \frac{\kappa L}{u_0 \bar{U}} (\bar{\alpha}_i - \alpha_t) I_i \tag{17}$$

where κ is the speed of light, α_t the threshold gain, and the bar indicates values averaged over the lateral distance.

The threshold gain is given by

$$\alpha_t = -(\ell n r_1 r_2)/2Z \tag{18}$$

where r_1 and r_2 are the mirror reflectivities and Z the gain length, which is proportional to L. When the pressure-geometry scaling condition given by Eq. (12) is applied, and with ρ being simply proportional to p, it is seen that α_t and α both vary with ρ_0. Combination of Eqs. (10), (14), (15), and (17) then indicates that I varies with ρ_0 and that all similarity conditions for the species transport equation are satisfied. Under the indicated conditions that α and I both vary with ρ_0, Eqs. (9) and (13) show that similarity conditions for the energy equation are also satisfied.

Therefore, complete flow, chemical, and radiative similarities are achievable by employing pressure and geometrical size scaling as indicated by Eq. (12). The only significant requirements for precise scaling are that the dominant chemical reactions are bimolecular and that pressure is low (i.e., lasing occurs in Doppler or velocity broadened systems where Lorentz or pressure broadening is relatively unimportant). These conditions are indeed realized in a wide variety of chemical lasers such as HF and DF.

Performance Scaling

The performance parameters discussed here are specific power σ, power flux δ, and lasing mode length $2X_c$. The power per unit mass flow is

$$\sigma = \sum_i \int_0^x \bar{\alpha}_i I_i A \, dx \Big/ \int_0^A \rho u \, dA \tag{19}$$

where A is the cross-sectional area. The power per unit area is

$$\delta = \sum_i \int_0^x \bar{\alpha}_i I_i \, dx \tag{20}$$

The length $2X_c$ is defined by the point where power has reached the maximum value.

Applying the scaling relations of $\rho \sim L^{-1}$, $\bar{\alpha} \sim \rho$, $I \sim \rho$, $A \sim L^2$, and $x \sim L$, it is seen that σ is independent of scale while $\delta \sim L^{-1}$ and $2X_c \sim L$. These results indicate that σ and the product of δ and $2X_c$ can be maintained unaltered between hardware sets of different sizes. This implies that once a configuration is designed to yield a desired level of σ, a tradeoff between δ and $2X_c$ can be made by simply changing the size of the hardware while keeping the product $p_0 L$ constant.

Discussion

The scaling analysis is applicable to transient, three-dimensional, and/or turbulent flow. The time in the transient terms scales as $t \sim x/u$, and hence the time scale is simply proportional to L. For three-dimensional flows, the additional terms do not introduce any new scaling factors. Also, since the equations considered are of Navier-Stokes form, the scaling results are applicable to recirculating as well as mixing layer type flows. For turbulent flows, the effective viscosity varies as $\rho_0 u_0 L$, while the Prandtl and Schmidt numbers are independent of ρ and L. Since $\rho_0 \sim L^{-1}$, turbulent flows scale in the same manner as laminar flows.

The pressure-geometry scaling is applicable to nozzle flows as well as cavity flows, since similarity in boundary conditions can easily be shown to be satisfied. In particular, consider chemical deactivation along a wall. The species boundary condition there is

$$\frac{\partial c_i}{\partial \eta} = -\frac{\rho L S c_i \Gamma_i c_i}{\mu} \left(\frac{R_i T}{2\pi} \right)^{1/2} \quad \text{at wall} \tag{21}$$

where Γ and R denote deactivation efficiency and gas constant, respectively. Under common conditions, Γ is nearly independent of pressure, and Eq. (21) with $\rho \sim L^{-1}$ indicates that similarity prevails.

The performance scaling relations of the present study have been verified by numerical solution of the governing

equations. The computation was performed for a source flow mixing layer type DF chemical laser cavity using a stable implicit numerical procedure.[4] Seven vibrational levels of DF, each having 14 equilibrium rotational states, were considered. The chemical reactions were taken from Ref. 5, which are predominantly bimolecular. Three cases have been computed for illustration. The reference case corresponds to a cavity inlet pressure of $p_0 = 4.66$ Torr and an element length (centerline to centerline distance between a pair of fuel and oxidizer nozzles) of $L = 0.198$ cm. The second case is for $2p_0$ and $0.5L$, and the third for $0.5p_0$ and $2L$.

The results show that the specific power σ of the second and third cases $= 0.991$ and 1.008, respectively, of the reference case. Hence specific power is essentially unaltered by the scaling. The small differences are due to pressure broadening effects, which are indeed minor.

The power flux distributions, normalized by the maximum value reached in the reference case, are shown in Fig. 1. It is seen that the power flux scales inversely with L, while the power cutoff distance scales directly with L.

The present analysis indicates that, even though laser flowfields are quite complicated and involve various nonlinear physical processes, unique and precise pressure-geometry scaling results can be reached. The restrictions, which have been delineated at the relevant points of the analysis, are surprisingly few and are easily satisfied under common operating conditions. Specifically, for combustor temperatures above ~ 1500 K, the effect of pressure variation on incomplete combustion is small. For cavity pressures under ~ 20 Torr, three-body reactions and pressure-broadening effects on gain are negligible.

Conclusions

In conclusion, the results of the present scaling analysis indicate that:

1) The conservation equations and constitutive relations for chemical laser flows are satisfied by the pressure-length scaling.

2) Flow similarity is achieved due to invariance in Reynolds number. Chemical similarity is obtained for bimolecular reactions. Radiative similarity is obtained as gain and intensity both vary with density.

3) Tradeoff between power flux and lasing mode length can be accomplished at a given specific power level by applying pressure-geometry scaling.

References

[1]Bird, R. B., Stewart, W. E., and Lightfoot, E. N., *Transport Phenomena*, John Wiley and Sons, New York, 1960.

[2]Gosman, A. D., Pun, W. M., Runchal, A. K., Spalding, D. B., and Wolfshtein, M., *Heat and Mass Transfer in Recirculating Flows*, Academic Press, New York, 1969.

[3]Thoenes, J., Hendricks, W. L., Kurzius, S. C., and Wang, F. C., "Advanced Laser Flow Analysis (ALFA) Theory and User's Guide," Lockheed Missiles and Space Co., Rept. AFWL-TR-78-19, Feb. 1979.

[4]Quan, V., Persselin, S. F., and Yang, T. T., "Computation of Reacting Flowfield with Radiation Interaction in Chemical Lasers," *AIAA Journal*, Vol. 21, Sept. 1983, pp. 1283-1288.

[5]Cohen, N. and Bott, J. F., "Review of Rate Data for Reactions of Interest in HF and DF Lasers," Aerospace Corp. Rept., TR-0083(3603)-2, 1982.

Numerical Evaluation of Line Integrals with a Logarithmic Singularity

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Introduction

In solving two-dimensional boundary value problems by the boundary element method, line integrals of the following form are encountered[1-3]

$$\int_C \varphi(s)K(r)\mathrm{d}s_q \tag{1}$$

where C is the boundary element, i.e., a finite plane arc; $r = r(p,q) = |p-q|$ is the distance between any two points p and $q \in C$; the index q in the arc length element $\mathrm{d}s_q$ indicates that point q varies during integration while the point p remains constant; $\varphi(s)$ is a continuous function of the arc length $s = s(p,q)$ measured from point p; and the kernel function $K(r)$ has a logarithmic singularity. That is,

$$K(r) = \psi_1(r) + \psi_2(r)\ell nr \tag{2}$$

The functions $\psi_1(r)$ and $\psi_2(r)$ are continuous. In mechanics, integral (1) represents a single-layer potential due to a material curve C of line density $\varphi(s)$ when the field point p is on the source.

For computer implementation of the boundary element method, the boundary element is approximated by a straight line or a parabolic arc.[3] In the first case, $r = s$ and integral (1) may be evaluated by either analytical integration, when the functions $\varphi(s)$, $\psi_1(r)$, and $\psi_2(r)$ are simple, or numerically using a special logarithmically weighted integration formula[3,4] when these functions are not simple. However, when the element is approximated by a higher-order curve, the evaluation of line integral (1) is more difficult. In this case, the integral has been evaluated by the method of subtracting the singularity.[5] In this method, a function is subtracted from the integrand that has a singularity of the same kind and can be integrated formally. This method has been employed in Ref. 6 for the numerical evaluation of integral (1) using the actual geometry of the element.

In this Note, a simple method is presented for the evaluation of line integral (1) by removing the singularity by means of integrating by parts when the curved element is approximated by a parabolic arc. This procedure can be easily programmed on a computer.

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